The Value of Capacity Information in Supply-Chain Contracts

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Helpful comments were received from Dennis Caplan, John Hassell, and Brian Mittendorf and workshop participants at Oregon State University.
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1 Introduction

One of the primary challenges facing many firms is ensuring a sufficient quantity of productive inputs. This challenge is particularly acute when the firm must compete with other consumers of the input and when the supplier has significant market power. In this paper, we examine a firm that desires to acquire an essential input from a supplier. The supplier, in our model, faces capacity constraints and must allocate capacity to produce for this firm and to produce for an external market. In the absence of strategic supply chain management, the purchasing firm would necessarily take a hit-or-miss approach to acquiring this product. They would either rely on the supplier to offer a quantity and price or they might make an offer to purchase. In the current supply chain environment, supply chain partners, as these firms are, share information about their operations. It is this information sharing that is the focus of our paper.

Specifically, we compare three different contract structures for the purchaser and supplier: (1) an environment in which the purchaser makes a take it or leave it offer to the supplier, (2) an environment in which the two entities share information, and (3) an environment in which there is no information asymmetry. The first case might arise if the purchaser knows the general nature of the supplier’s production environment, but is uncertain about the supplier’s capacity. In the second case, the supplier and purchaser enter into a relationship in which the purchaser commits to a quantity and price schedule in exchange for the sharing of capacity information that is known by the supplier. This situation might arise in an ongoing relationship between supply-chain partners. Finally, the third case implies that both the purchaser and supplier are fully informed about
production capacity. This last case exemplifies the best possible joint outcome for the two firms and might reflect the outcome if the firms merged into a single economic entity.

We find that active supply chain management, as reflected by our second case in which the two firms enter into an information sharing relationship, offers superior profits to both the supplying firm and the purchasing firm relative to the case in which the purchaser is uninformed about the supplier’s production capacity. In addition, we find that these benefits are more pronounced if the supplying firm faces significant competitive pressure in the outside market for its product. The contract in which both contracting parties are fully informed yields a higher gross payoff and the purchasing firm obtains a higher payoff, but the payoffs to the supplying firm are diminished relative to the contract with information sharing.

Our model also yields some interesting empirical predictions. First, we predict, based upon our results, that information sharing will lead to more long-term supply-chain relationships in environments in which firms face more intense market pressure for their products. In addition, we believe that in vertical-integration acquisitions, more acquisition gains will be paid to target shareholders when the target firm has relatively more market power or when the value of the product to the purchaser is relatively high.

Our paper relates to a growing literature in supply chain contracting. Baiman and Rajan (2002a) provide a useful overview of the relationship between contract structures and the inter-firm incentive issues and relationships that frequently arise in supply chain partnerships. They note that incomplete contracts are frequently more descriptive of perpetual relationships in which many contract details are not resolved in a single period,
but that some important supply chain contract issues, such as whether to outsource and how to organize suppliers, can legitimately be examined in a complete contracting setting. As our paper deals primarily with information sharing in procurement, we believe that complete contracts are appropriate for our analysis.

Baiman and Rajan (2002b) examine how information sharing may lead to opportunism on the part of the previously uninformed party. In their model, the buyer can invest in innovation and provide private production information to the supplier that the supplier can possibly exploit. However, they do not compare their setup to a straightforward, arms-length transaction, which is the primary focus of our paper.

Baiman et al. (2000) examine a buyer-supplier relationship where the quality of the product is the basis of the contract. Their analysis examines a double moral-hazard setting where the supplier incurs unobservable quality costs and the buyer incurs unobservable quality appraisal costs. Essentially, the two parties share the responsibility for providing a quality product to the market. They further provide some insights into information sharing that can reduce the inefficiency caused by the information asymmetry. These results were extended in Baiman et al. (2001). In this study, the authors examine the balanced scorecard issue of measuring internal processes by contracting internal and external failure rates.

Narayanan et al. (2005) and Arya and Mittendorf (2007) examine issues similar to that in our paper in the sense that both of these papers examine a tension between alternative uses of production capacity. Narayanan et al. (2005) examine the benefit of supplier insurance in an environment in which two purchasers compete for consumers. They demonstrate that by subsidizing leftover inventory, they can reduce the intensity of
competition between buyers and, therefore, maintain a higher market price for the product. Our model incorporates a similar feature. By dividing capacity between sales to the purchaser (through a single contract) and to the remaining market, the supplier can maintain a higher overall price for his production. Of course, we examine the purchaser’s problem of procurement rather than the supplier’s problem of revenue maximization. In this sense, our paper more resembles Arya and Mittendorf (2007) who examine the interplay between internal procurement and external procurement. They demonstrate that the agency costs inherent in internal procurement can improve the negotiating position of the procuring firm when negotiating with an external source. We examine the opposite problem. In our model, the purchaser is competing for capacity with external purchasers. But in both models, the purchaser is a lesser-informed party in the procurement contract.

The remainder of this paper is organized as follows. Section 2 will describe and characterize the product procurement problem facing the purchaser and characterize the contract without information sharing. In section 3, we describe and characterize the contracting relationship with information sharing that we refer to as active supply chain management. In section 4, we describe and characterize the Full Information contract in which the purchaser is able to costlessly learn the capacity of the supplier. We compare the three different information structures in terms of the quantity of goods sold to the purchaser and the price that the purchaser pays for these goods and in terms of the expected payoffs to the purchaser and supplier in section 5. We provide concluding remarks in section 6.
2 Product procurement problem

Consider a firm that relies on a critical component or product that is produced by a single firm. The supplying firm has limited capacity to produce this product and the purchasing firm must compete for these capacity resources. We begin by describing the production and marketing environment facing the supplier without consideration of the sales to the purchaser.

The supplier has capacity of $K$ for producing the product, but $K$ is privately observed. The purchaser knows only the distribution of possible values of $K$: $K \sim U[0, \bar{K}]$.\(^1\) We assume that the supplier faces a linear demand for their product. Specifically, we assume the price, $P(x)$ that their product brings, as a function of the number of units ($x$) that are sold, is given by the following linear demand function.

$$P(x) = A - bx.$$ \hfill (1)

As each unit has a cost of $c$ to produce, the profit earned from these sales, $\Pi_s(x)$, is the total revenue less the cost of production.

$$\Pi_s(x) = x(A - bx) - cx.$$ \hfill (2)

In the absence of any capacity considerations, the firm would maximize profits by producing and selling $x^*$ units where

$$x^* = \frac{(A - c)}{2b}.$$ \hfill (3)

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\(^1\) While each of our models can be characterized for more general density functions, the assumption that capacity is uniformly distributed over a closed interval makes the comparison across different information structures more transparent and improves the intuition.
Given capacity constraints, though, the supplier will produce and sell all $K$ units unless $K > \frac{(A-c)}{2b}$, which we will denote as $K_1$. In that case, the supplier will produce

$$K_1 = \frac{(A-c)}{2b}$$

units and will have excess capacity of $K - K_1$.

The purchaser desires to purchase as many units as possible in a negotiated sale with the supplier and obtains a benefit (either from external sales or from further converting the product) of $V$ per unit purchased. We assume, in this setting, that the purchaser offers to purchase $y$ units of the product for a price of $T$. If the supplier’s profit is increased by the offer, then the offer will be accepted. Otherwise the supplier will reject the offer.\(^2\) We refer to this negotiated agreement as the No Information contract.

If the supplier is operating at capacity ($K < K_1$), then if it provides the purchaser with $y$ units, profit of $y(A - b(2K - y))$ will be foregone. This amount represents the difference between the revenue that the supplier earns for manufacturing and selling $K$ units and for manufacturing and selling $K - y$ units. Since the supplier will be selling the remaining units to the purchaser, the production costs will be $c^*K$. If, on the other hand, the supplier is operating below capacity ($K \geq K_1$), then profit of

$$\frac{\left(A - c - 2b(K - y)\right)^2}{4b} + cy$$

will be foregone.

\(^2\) There are many different possible negotiation equilibria that we could consider. We assume for simplicity that the uninformed party makes a take it or leave it offer to the informed party that is either accepted or rejected. Since the supplier has an informational advantage, any recursive offer setting would result in the same outcome unless the purchaser has a credible threat to cease negotiations or unless negotiations might stochastically cease.
The purchaser must make an offer that compensates the supplier for the lost profit in order for the supplier to accept. The purchaser chooses $y$ and $T$ to maximize the expected payoff below.

$$\left(1 - G\left[K^c\right]\right)\left(V_{y^{NI}} - T^{NI}\right),$$

(4)

where $K^c$ satisfies:

$$K^c = \frac{\left(A + by^{NI}\right)}{2b} - \frac{T^{NI}}{2by^{NI}}.$$  

(5)

$K^c$ defines the point where the payment of $T$ just compensates the supplier for the opportunity cost of accepting the offer. For $K \geq K^c$, the supplier will accept the offer of $(y^{NI}, T^{NI})$ and for all $K < K^c$, the supplier will reject the offer. The purchaser must balance the cost of the purchase ($T$) against the likelihood that the offer will be rejected. Proposition 1 characterizes the offer in the No Information contract.

**Proposition 1.** In the No Information contract, the purchaser makes a take it or leave it offer to the supplier for $y^{NI}$ units at a price of $T^{NI}$, where:

$$y^{NI} = \frac{1}{3b}\left(V - A + 2bK\right),$$

(6)

and

$$T^{NI} = \frac{1}{9b}\left(V - A + 2bK\right)\left(2V + A - 2bK\right).$$

(7)

The supplier will accept the offer whenever

$$K \geq K^c = \frac{A + 4bK - V}{6b}.$$  

(8)

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3 The superscript NI denotes these values as No Information contract values.

4 This value compensates the supplier if it is operating at capacity. If the supplier is operating below capacity ($K > K_1$), its opportunity cost would be lower and would require a lower payment. However, the purchaser, in expectation, is never better off choosing that lower payment because the offer will be rejected much more frequently. A proof of this point is available from the authors.
and reject the offer otherwise.  

(proof in appendix.)

This strategy is hit or miss and the offer will only be accepted with probability, \( (1 - G[K^e]) = (\bar{K} - K^e) / \bar{K} \). The purchaser can improve the likelihood of the offer being accepted by increasing the payment offer \( T \) or by decreasing the quantity requested \( y \) (while continuing to offer to pay \( T \)). Note that if \( T \) increases or if \( y \) decreases, the value of \( K^e \) in (5) decreases because the offer is a better deal for the supplier. And as \( K^e \) decreases, \( (1 - G[K^e]) \) increases. In maximizing (4), the purchaser would prefer to choose a higher value of \( y \) and a lower value of \( T \) in order to obtain the most product for the least money, but simultaneously wishes to increase the likelihood that the offer will be accepted and this implies a lower value of \( y \) and/or a higher value of \( T \). The purchaser balances these tensions to arrive at the optimum offer pair, \( (y^{NI}, T^{NI}) \). In the next section, we introduce a contracting solution to the purchaser’s problem in which the supplier reports its capacity in exchange for a purchase commitment by the purchaser. This solution to the procurement problem is consistent with the information sharing characteristics of supply chain management.

3 Supply chain management

We now consider a contract in which the purchaser offers a contract to the supplier with the following characteristics. The contract specifies a payment \( T \) for each possible transfer of \( y \). Then when the supplier learns the value of \( K \), it reports a value \( \hat{K} \) to the purchaser. The purchaser then pays the contract price and the supplier provides the
contract amount of the product \( y \). Since there is no monitoring, the truthfulness of the report \( \hat{K} \) must be self-enforcing.

Before proceeding to purchaser’s problem, we first note that the supplier earns economic rents from its superior information. These rents, which will be useful in characterizing the contracting problem facing the purchaser, are defined below.

\[
r(\hat{K} | K) = T(\hat{K}) + \Pi_s(K - y(\hat{K})) - c^*y(\hat{K}) - \Pi_s(K),
\]

(9)

where \( \Pi_s(\cdot) \) is defined by (2). This expression recognizes the fact that the value of the information held by the supplier equals the difference between its total profit under the contract (profit on outside sales plus the profit from the sale of \( y \) units to the purchaser) and the profit that it would have earned had it sold exclusively to external customers.

The term, \( r(\hat{K} | K) \), reflects the fact that the rents depend upon the reported capacity level, \( \hat{K} \), as well as the actual value of \( K \). Note that both \( T(\hat{K}) \) and \( y(\hat{K}) \) depend upon the reported value, \( \hat{K} \).

We now describe the purchaser’s contracting problem in the following constrained optimization program.

\[
\text{Maximize} \quad M = \int_0^{\overline{K}} \left[ Vy(k) - T(y(k)) \right] \frac{1}{\overline{K}} dk
\]

(10)

Subject to:

\[
r(K) = r(K | K) \geq 0 \quad \forall K \in \left[ 0, \overline{K} \right]
\]

(11)

\[
r(K) \geq r(\hat{K} | K) \quad \forall \hat{K} \in \left[ 0, \overline{K} \right], K \in \left[ 0, \overline{K} \right], \hat{K} \neq K
\]

(12)

Constraint (11) requires that \( r(K) \), which is the economic rent of reporting \( \hat{K} = K \) when \( K \) is observed, is weakly positive. In other words, the supplier’s total profit (from
external sales and from sales to the purchaser) for all possible values of $K$ must be at least as high as if it just sold externally.

Constraint (12) requires that for every possible observation of $K$, there is no announcement, $K' \neq K$, such that the total profit to the supplier is strictly higher by reporting $K'$ than it is if it reports honestly ($K$). For expositional purposes, we refer to the solution to this program as the Supply Chain contract and will employ the superscript, SC.

We now characterize the solution to Supply Chain contract described in expressions (10), (11), and (12). The supplier observes $K$ and reports $\hat{K}$. The Truth-Telling constraint (constraint (12)) requires that, given the supplier observes $K$, the truthful report $\hat{K} = K$ yields a higher payoff than any possible dishonest report $\hat{K} \neq K$. Algebraically, this requires that the $(y^{SC}(\hat{K}), T^{SC}(\hat{K}))$ contract offered by the purchaser makes $\hat{K} = K$ at least weakly optimal for the supplier.

Again, we must break the problem into two cases: $K \leq K_1$ and $K > K_1$. The supplier desires to maximize the following condition by its choice of $\hat{K} \in [0, \bar{K}]$:

\[
r(\hat{K} \mid K) = T\left( y(\hat{K}) \right) + A \left( K - y(\hat{K}) \right) - b \left( K - y(\hat{K}) \right)^2 - cK - \Pi_s(K).
\]

For each $\hat{K}$ that the supplier reports, the purchaser requests a contracted amount of $y$ and gives the supplier a contracted price, $T$. The first component of (13) is the price. The second two components combine to yield the revenue from outside sales that the supplier will earn on whatever capacity remains after providing the sale of $y(\hat{K})$ units to the purchaser. If the supplier is producing at capacity, then it incurs a production cost of
Finally, the rent payment that the supplier is attempting to maximize is the difference between these two sources of profit and the profit that it would have earned in the absence of sales to the purchaser, \( \Pi_s (K) \). If \( K \leq K_1 \), then this reservation profit is

\[
AK - bK^2 - cK
\]

and if \( K > K_1 \), it is \( \frac{(A - c)^2}{4b} \).

Proposition 2 characterizes the equilibrium contract. Though \( \Pi_s (K) \) changes from the interval in which \( K \leq K_1 \) to that in which \( K > K_1 \), the transfer amount, \( y^{sc} (\hat{K}) \) and the payment, \( T^{sc} (\hat{K}) \) do not change at this cutoff point.

**Proposition 2.** The equilibrium contract in which the supplier truthfully communicates capacity to the purchaser is characterized by the following transfer amount, \( y^{sc} (\hat{K}) \) and payment, \( T^{sc} (\hat{K}) \).

\[
y^{sc} (\hat{K}) = 2\hat{K} - \bar{K} - \frac{A - V}{2b}
\]  

as long as \( K \) is greater than or equal to the average of \( \bar{K} \) and \( (A - V) / 2b \). Otherwise, no units will be sold to the purchaser. The payment to the supplier is computed as:

\[
T^{sc} (\hat{K}) = \frac{1}{8b} \left( 3V + A + 2b \left( 2\hat{K} - 3\bar{K} \right) \right) \left( V + 2b \left( 2\hat{K} - \bar{K} \right) - A \right).
\]  

(proof in appendix)

The supplier will provide some units of \( y \) to the purchaser for a broader range of capacity levels under the Supply Chain contract than under the No Information contract. In other words, the critical value of \( K^c \) in (5) is greater than \( \frac{1}{2} \left( \frac{(A - V)}{2b} + \bar{K} \right) \). On the other hand,
at \( K = K^c \), the supplier will sell more units to the purchaser under the No Information contract than under the Supply Chain contract. In fact, at \( K = K^c \), \( y^{NI} = 2 \cdot y^{SC}(K^c) \).

In the next section, we turn to a Full Information contract in which the supplier and purchaser are equally well informed prior to contracting. This case reflects the upper bound of information sharing. This setting might reflect the maximum informational benefit that could be obtained through vertical integration.

4 Full Information contract

In this contract, both the supplier and the purchaser observe the value of \( K \). The purchaser then offers to purchase \( y \) units from the supplier for a price, \( T \). \( T \) must provide the supplier with sufficient compensation to cover the opportunity cost of not selling those units externally, but does not include any information rents. In this case, the contract will differ for situations in which \( K \leq K_1 \) and for those in which \( K > K_1 \). If \( K \leq K^1 \), then the supplier’s opportunity cost is given by the following condition:

\[
\Pi_s(K) = AK - bK^2 - cK
\]

which implies:

\[
T - y(A - b(2K - y)) \geq 0 .
\]

On the other hand, if \( K > K_1 \), the supplier’s opportunity cost is only:

\[
\Pi_s(K_1) = \frac{(A - c)^2}{4b}
\]

which implies:

\[
T + (K - y)(A - b(K - y)) - cK - \frac{(A - c)^2}{4b} \geq 0 .
\]
Since $K$ is common knowledge, the purchaser will know whether the price, $T$ is
constrained by (17) or by (19) and will choose the optimal values of $T$ and $y$ accordingly.
We will denote these optimal values with a superscript $FB$: $(y^{FB}, T^{FB})$. The purchaser
will choose $y$ to maximize $Vy - T$ where $T$ is the value that solves (17) or (19) as an
equality. Proposition 3 characterizes the optimal choices of the Full Information contract.

**Proposition 3**

If $K \leq K_1 = \frac{(A-c)}{2b}$, then the following choices define the contract between the
purchaser and the supplier under the Full Information contract.

\[
y^{FB} = K - \frac{A - V}{2b}
\]  \tag{20}

and

\[
T^{FB} = \frac{(A - 2bK + V)(-A + 2bK + V)}{4b}.
\]  \tag{21}

On the other hand, if $K > K_1$, then the following choices define the contract between the
purchaser and the supplier under the Full Information contract.

\[
y^{FB} = K - \frac{(A - V)}{2b}
\]  \tag{22}

and

\[
T^{FB} = \frac{V^2 - c(2A - c - 4bK)}{4b}.
\]  \tag{23}

(proof in appendix)

Note that the values of $y^{FB}$ in (20) and (22) are identical and that the supplier will
only sell to the purchaser when $K > \frac{(A - V)}{2b}$. The value of $T^{FB}$ in (23) is strictly greater
than the value in (21), but that is because more units are being sold. In fact, the unit price \((T^{FB} / y^{FB})\) is decreasing in \(K\) even though both (21) and (23) are increasing in \(K\).

5 Value of information sharing

In this section, we compare the different contracts in terms of the quantity of product that are sold to the purchaser, the price of the transaction, and the relative expected payoffs to the purchaser and supplier. We begin by comparing the amount of product that is sold to the purchaser. To do this, we first compute the capacity level under each contract below which no product is sold to the purchaser. We denote these values, \(K^{NI}\), \(K^{SC}\), and \(K^{FB}\), for the No Information contract, the supply-chain contract, and the Full Information contract respectively. Proposition 4 characterizes and compares these three values.

**Proposition 4**

In the No Information contract, no product is sold to the purchaser if:

\[
K \leq K^c = \frac{(A + 4b\bar{K} - V)}{6b}.
\]  

(24)

In the Supply chain contracting contract, no product is sold to the purchaser if:

\[
K \leq K^{SC} = \frac{A + 2b\bar{K} - V}{4b}.
\]  

(25)

In the Full Information contract, no product is sold to the purchaser if:

\[
K \leq K^{FB} = \frac{(A - V)}{2b}.
\]  

(26)

In addition, \(K^{FB} < K^{SC} < K^{NI}\).

*(proof in appendix)*
The Full Information contract allows the purchaser to find the price-quantity combination that just covers the loss of sales to the external market. In the Supply chain contract, the purchaser must cover these losses and must also provide the supplier information rents which imply that the balance between price and quantity is suboptimal and it lowers the amount purchased in equilibrium. Under the No Information contract, since it must offer a single quantity to purchase in a take it or leave it offer, the purchaser would not wish to purchase one of the lower amounts that characterize \( y^{SC} \left( K^{SC} \right) \) or even \( y^{FB} \left( K^{FB} \right) \). It would not be worth it. So the purchaser chooses a higher quantity and price combination. This implies that it will be accepted less often. As a result, \( K^{NI} > K^{SC} > K^{FB} \). Our next result, Proposition 5 characterizes the relative amount of product that is sold to the purchaser under the three possibilities.

**Proposition 5**

For all values of \( K \), \( y^{FB} > y^{SC} \) and \( y^{FB} \geq y^{NI} \). \( y^{FB} = y^{NI} \) if and only if \( K = K^c \). In addition, there exists a unique capacity level, \( K^{NI-SC} \), such that for all \( K \in \left( K^c, K^{NI-SC} \right) \), \( y^{NI} > y^{SC} \) and for all \( K < K^c \) or \( K > K^{NI-SC} \), \( y^{NI} < y^{SC} \). Finally, the offer of \( y^{NI} \) is greater than the expected quantity sold under the Full Information contract.

*(proof in appendix)*

The fact that the Full Information value of \( y \) is greater than either of the others is obvious. The fact that \( y^{FB} = y^{NI} \) when \( K = K^{NI} \) follows from the fact that if the purchaser guesses just right, then it obtains the Full Information contract pair. Finally, since \( y^{NI} \) is fixed in \( K \), it is larger than \( y^{SC} \) for lower values of \( K \) and smaller for higher values of \( K \).
The true comparison between the different contracts is in the expected payoff obtained by the supplier and the purchaser under each contract alternative. This result is presented in Proposition 6.

**Proposition 6**

The expected payoff to the purchaser is higher under the Full Information contract than under the Supply Chain Contract and higher under the Supply Chain Contract than under the No Information contract. The expected payoff to the supplier is higher under the Supply Chain contract than it is under either the Full Information contract or the No Information contract.

*(proof in appendix)*

The expected benefits to the purchaser are straightforward. As the purchaser obtains more information, it is more able to contract efficiently. It is somewhat surprising that we find a Pareto improvement when moving from the No Information contract to the Supply Chain contract. The expected payoffs of both firms improve because of the information sharing in the Supply Chain contract. This is not the case when moving from the Supply Chain contract to the Full Information contract. In that case, the supplier loses the information rents that accrue under the Supply Chain contract.

Our next result provides more intuition about the nature of the relationship between these contracts. This result is stated in Corollary 1.
Corollary 1

The improvement in expected payoffs for both parties when moving from the No Information contract to the Supply Chain contract is increasing in the degree of competition facing the supplier in the external market.

(proof in appendix)

We measure the degree of competition facing the supplier by the marginal decrease in demand, b, induced by an increase of one unit of sales. The larger the value of b, the more sensitive price is to the quantity demanded. As this value increases, the Supply Chain contract becomes increasingly attractive to both the supplier and the purchaser. As this value increases, the divergence in preferences regarding the Full Information contract also increases.

For empirical researchers, this result implies that long-term Supply Chain contracts that involve substantial information sharing are more likely to arise in settings that involve competitive pressures for the supplier. Even though the most obvious benefactor of these contracts is the purchaser who is attempting to procure the product, the extent to which these contracts arise may be driven more by the desires of suppliers to manage their capacity and maintain a premium price for their products.

We now consider the value to purchasers of vertical integration. We have already established that the purchaser enjoys strict improvements in expected payoffs when moving from a Supply Chain contract to a Full Information contract. This type of movement might represent the movement from supply chain partners to a single firm in which the purchaser acquires the supplier in order to improve product procurement. With our model, we can identify factors that might affect the value of the acquisition to the
acquiring firm and the factors that might drive the target firm to take steps to avoid acquisition. We provide results related to these factors in Corollary 2.

**Corollary 2**

If \( K_1 = \frac{A-c}{2b} \) is sufficiently high, then the value to the purchaser of acquiring the supplier in an informationally motivated vertical integration is increasing in the value of the product to the purchaser and is decreasing in the demand for the product in the external market.

*(proof in appendix)*

These results are opposite those for the supplier. As a result, we expect that as the product becomes relatively more valuable to the purchaser than it is to the supplier, the purchaser will be more inclined to acquire the supplier. Relative to the Supply chain relationship, though, the supplier will be less interested in such an acquisition. Therefore, in these cases, we expect that relatively more acquisition gains would accrue to target shareholders (the shareholders in the supplier).

**6 Conclusion**

We compare three contract forms between a supplier of a critical product and the purchaser of that product. A critical feature of our model is that the supplier has private information about production capacity and an external market for the product exists. The first contract is one in which the purchaser must make a take it or leave it offer of a quantity and price to the supplier, but is uniformed about the capacity. We find that the purchaser’s offer includes a relatively large request for quantity such that it is most frequently rejected. The second contract is one in which the supplier, in exchange for a purchase commitment by the purchaser, agrees to share capacity information. This
contract reflects the current practice of information sharing between supply chain partners. The supplier collects information rents in this contract, but it allows for a strict improvement in expected payoffs for both the supplier and the purchaser over the first contract. In addition, we find that the improvements for both the supplier and the purchaser are increasing in the extent to which suppliers face strong competition for their products in external markets. Finally, we consider a contract without information asymmetry. One motivation for vertical integration is to eliminate contracting frictions and to obtain a more secure source of products. The purchaser strictly prefers this relationship to either the information sharing relationship or the contract without information, but the supplier prefers the information sharing relationship since in this case, it must forego the information rents.

Our results motivate some interesting empirical questions. First, we expect that supply chain relationships will arise more frequently when suppliers face strong competition for their products in other markets. In addition, we expect that if a purchasing firm acquires a supplier firm in an attempt to improve procurement, that the share of acquisition gains that are allocated to target shareholders (the shareholders of the supplier firm) are likely to be increasing in the value and importance of the product to the purchaser and decreasing in the extent of competition that the supplier faces in the external market.
References


Appendix

Proof of Proposition 1

We begin by examining the supplier’s payoff for accepting an offer \((y^NI, T^NI)\). There are two discrete possibilities for this payoff that depend upon whether the supplier’s capacity is above or below \(K_1\), the capacity level above which profit on external sales decreases.

We will denote these distinct offer pairs as \((y^NI_0, T^NI_0)\) and \((y^NI_1, T^NI_1)\). It will turn out that only one of these offers (the first one) will be chosen in equilibrium. If the supplier’s capacity is at or below \(K^1\), then the payoff to the supplier of accepting the contract is:

\[
T^NI_0 - y^NI_0 \left( A - b \left( 2K - y^NI_0 \right) \right). \tag{27}
\]

Since \(y^NI_0\) and \(T^NI_0\) are fixed, the supplier is (weakly) better off if it accepts the contract if and only if:

\[
K \geq K^c_0 = \frac{(A + by^NI_0)}{2by^NI_0} - \frac{T^NI_0}{2by^NI_0}. \tag{28}
\]

If the supplier’s capacity is above \(K_1\), then the payoff to the supplier of accepting the contract is:

\[
\frac{1}{4b} \left( 2A \left( c + 2b \left( K - y^NI_1 \right) \right) + 4b \left( T^NI_1 - b \left( K - y^NI_1 \right)^2 \right) - \left( A^2 + 4bcK + c^2 \right) \right). \tag{29}
\]

The supplier will accept the contract if and only if:

\[
K \geq K^c_1 = y^NI_1 + \frac{A - c - \sqrt{b(T^NI_1 - cy^NI_1)}}{2b}. \tag{30}
\]

The purchaser, therefore, has two potential choices to pursue, to choose an offer pair, \((y^NI_0, T^NI_0)\), that the supplier will accept whenever \(K \geq K^c_0\) or to choose an offer pair,
that the supplier will accept whenever $K \geq K_c^c$. We will analyze the
purchaser’s problem in both cases and then compare its expected payoff from the two
choices. The purchaser’s expected payoff from offering \((y_0^{NI}, T_0^{NI})\) is

\[
\Pi_p = (1 - G[K_0^c]) (V y_0^{NI} - T_0^{NI}).
\]  

(31)

The first order conditions for maximizing this payoff are:

\[
\frac{\partial \Pi_p}{\partial y_0^{NI}} = V (1 - G[K_0^c]) - g[K_0^c] \frac{\partial K_0^c}{\partial y_0^{NI}} = 0 \quad \text{and}
\]

\[
\frac{\partial \Pi_p}{\partial T_0^{NI}} = - (1 - G[K_0^c]) - g[K_0^c] \frac{\partial K_0^c}{\partial T_0^{NI}} = 0.
\]  

(32)

(33)

Using the value of $K_0^c$ in (28), we can compute $\frac{\partial K_0^c}{\partial y_0^{NI}}$ and $\frac{\partial K_0^c}{\partial T_0^{NI}}$:

\[
\frac{\partial K_0^c}{\partial y_0^{NI}} = \frac{1}{2} + \frac{T_0^{NI}}{2b(y_0^{NI})} \quad \text{and}
\]

\[
\frac{\partial K_0^c}{\partial T_0^{NI}} = - \frac{1}{2by_0^{NI}}.
\]  

(34)

(35)

Substituting (34) and (35) into (32) and (33) implies:

\[
T_0^{NI} = y_0^{NI} (V - by_0^{NI}).
\]  

(36)

Substituting (36) into (28) yields a value of $K_0^c$ that is free of $T_0^{NI}$:

\[
K_0^c = y_0^{NI} + \frac{(A - V)}{2b}.
\]  

(37)

Substituting (37) into the purchaser’s payoff function in (31) yields:

\[
\Pi_0^{p} = (y_0^{NI})^2 2b (\bar{K} - y_0^{NI}) - (A - V) \frac{2K}{2\bar{K}}.
\]  

(38)
The first order condition for $y_0^{NI}$ is

$$\frac{\partial \Pi^p}{\partial y_0^{NI}} = y_0^{NI} \left( \frac{2b\bar{K} - 3by_0^{NI} - (A - V)}{\bar{K}} \right) = 0 \quad (39)$$

which implies

$$y_0^{NI} = \frac{V + 2b\bar{K} - A}{3b} \quad (40)$$

and

$$T_0^{NI} = \frac{(V + 2b\bar{K} - A)(A + 2V - 2b\bar{K})}{9b} \quad (41)$$

which are the values for $(y^{NI}, T^{NI})$ in Proposition 1. Substituting $(y_0^{NI}, T_0^{NI})$ into (38), we obtain the following value for $\Pi^p_0$:

$$\Pi^p_0 = \frac{(V + 2b\bar{K} - A)^3}{54b^2\bar{K}}. \quad (42)$$

Next, we must consider the possibility that the purchaser chooses instead to offer $(y_1^{NI}, T_1^{NI})$. The approach to characterizing these strategies is identical to that for $y_0^{NI}$ and $T_0^{NI}$ above. The strategies, in this case, are given by:

$$y_1^{NI} = \frac{3V + 4b\bar{K} - 2A - c}{8b} \quad \text{and} \quad T_1^{NI} = \frac{2V^2 - c(V + A - 4b\bar{K} - c)}{8b}. \quad \text{Substituting these values into (30) we obtain the following value for } K_1^c:\n
$$K_1^c = \frac{(2A - V - c + 4b\bar{K})}{8b}. \quad (43)$$

The expected payoff for the purchaser is computed as:

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5 Details are available from authors.
\[ \Pi_i^p = \frac{(V - c)(V - 2A + 4b\bar{K} + c)^2}{64b^2\bar{K}}. \]  

(44)

Subtracting \( \Pi_i^p \) in (44) from \( \Pi_0^p \) in (42), we obtain a value for the incremental expected payoff if the purchaser offers \( (y_0^{NL}, T_0^{NL}) \) relative to \( (y_1^{NL}, T_1^{NL}) \). This incremental expected payoff is

\[ D = \frac{(V + 2A - 3c - 4b\bar{K})^2(5V - 8A + 3c + 16b\bar{K})}{1728b^2\bar{K}}. \]  

(45)

If \( D > 0 \), then \( (y_0^{NL}, T_0^{NL}) \) is strictly preferred to \( (y_1^{NL}, T_1^{NL}) \). \( D > 0 \) if and only if

\[ (5V - 8A + 3c + 16b\bar{K}) > 0. \]  

(46)

Expression (46) is satisfied, if and only if \( b > \frac{8A - 3c - 5V}{16\bar{K}} \). Suppose that \( b < \frac{8A - 3c - 5V}{16\bar{K}} \). Since \( K_1^c \) is decreasing in \( b \), this implies that

\[ K_1^c > \bar{K} \frac{16A - 9V - 7c}{16A - 10V - 6c} > \bar{K}. \]  

(47)

As \( \bar{K} \) is the upper bound of the distribution, (47) is impossible. It must be the case, therefore, that (46) is satisfied and \( (y_0^{NL}, T_0^{NL}) \) given in (40) and (41). QED

**Proof of Proposition 2**

We begin by computing the price, \( T(y(K)) \), that is paid to the supplier for supplying \( y(K) \) units to the purchaser when observing capacity \( K \) and reporting capacity, \( \hat{K} = K \) for all capacity levels where the supplier has no idle capacity, \( K \leq K_1 \).

The supplier’s rent in (9) is given by:

\[ r(\hat{K}|K) = T(y(\hat{K})) + A(K - y(\hat{K})) - b(K - y(\hat{K}))^2 - cK - \Pi(K). \]  

(48)

Incentive compatibility requires:
\[ r_i(\hat{K} \mid K) = \frac{\partial r(\hat{K} \mid K)}{\partial K} = 0. \]  

(49)

where \( r(\hat{K} \mid K) \) denotes the report, \( \hat{K} = K \) when the supplier observes \( K \).

As this holds for any capacity level, \( K \), the total derivative of \( r(\bullet) \) with respect to the report and \( K \) is: \( r'(K) = r_1(K \mid K) + r_2(K \mid K) = r_2(K \mid K) \). This expression states that the changes in the supplier’s rent as a function of changes in both the report when the supplier is reporting honestly (the first \( K \)) and the actual \( K \) that is observed is equal to the sum of the partial derivatives of the rent with respect to the honest report and with respect to the observed \( K \). Since the supplier is reporting honestly (\( \hat{K} = K \)), we can simplify the notation to \( r'(K) \). In addition, as a result of our truth-telling constraint, (49), this simplifies to the partial of the rent with respect to the actual \( K \). As a result,

\[ r'(K) = -2b(K - y(K)) - c - \Pi'(K) = 2by(K). \]  

(50)

In order to obtain an expression for \( r(K) \), we integrate (50).

\[ r(K) = \int_{K}^{\hat{K}} 2by(k) dk + r(0) = \int_{K}^{\hat{K}} 2by(k) dk. \]  

(51)

Next, we substitute (51) into (48) to obtain:

\[ T(y(K)) = \int_{K}^{\hat{K}} 2by(k) dk + Ay(K) - bK^2 + b(K - y(K))^2. \]  

(52)

Next, we compute \( T(y(K)) \) for capacity levels \( K > K' \). Again,

\[ r'(K) = r_2(K \mid K) = A - 2b(K - y(K)) - c - \Pi'(K). \]  

(53)

In this case, though, \( r'(K) = A - 2b(K - y(K)) - c. \)  

(54)

\[ r(K) = \int_{K_i}^{\hat{K}} (A - 2b(k - y(k)) - c) dk + r(K') \]

\[ = \int_{K_i}^{K} (A - 2bk - c) dk + \int_{K}^{K_i} 2by(k) dk + \int_{0}^{K} 2by(k) dk \]

\[ = AK - bK^2 - cK - (AK_i - bK_i^2 - cK_i) + \int_{0}^{K} 2by(k) dk \]

\[ = AK - bK^2 - cK - \left(\frac{(A - c)^2}{4b}\right) + \int_{0}^{K} 2by(k) dk. \]  

(55)

Substituting for the payment:

\[ T(y(K)) = \int_{K}^{\hat{K}} 2by(k) dk + Ay(K) - bK^2 + b(K - y(K))^2. \]  

(56)
Although the form of the supplier’s rent differs between the cases when $K \leq K_1$ and $K > K_1$, the form of the payment is the same.

Substituting into the purchaser’s objective function for this payment, the purchaser’s problem becomes:

$$
\max_{y(K)} \int_0^K \left\{ V_y(K) - Ay(K) - b(K - y(K))^2 - \int_0^K 2by(k)dk + bK^2 \right\} \frac{1}{K} dK.
$$

Integrating by parts, (57) becomes:

$$
\max_{y(K)} \int_0^K \left\{ V_y(K) - Ay(K) - b(K - y(K))^2 - 2by(K)(\tilde{K} - K) + bK^2 \right\} \frac{1}{K} dK.
$$

Point-wise optimization provides the first order condition:

$$
V - A + 2b(K - y(K)) - 2b \left( \frac{1 - G[K]}{g[K]} \right) = 0,
$$

which implies:

$$
y(K) = K - \frac{A - V}{2b} - \frac{(1 - G[K])}{g[K]}.
$$

Substituting (60) into (56), we obtain the equilibrium value of $T(y(K))$ in (15).

QED

**Proof of Proposition 3**

If $K \leq K^1$, then the supplier’s reservation profit is given by (16) and the price must satisfy (17).

The purchaser then chooses $y$ to maximize:

$$
V_y - T = y(V - A + b(2K - y)).
$$

The purchaser therefore chooses

$$
y^* = K - \frac{A - V}{2b}
$$

and

$$
T^* = \frac{(A - 2bK + V)(-A + 2bK + V)}{4b}.
$$
The purchaser’s profit, as a function of $K$, is computed as:

$$V_y^* - T^* = \frac{(A - V - 2bK)^2}{4b}. \quad (65)$$

If $K > K_1$, then the supplier’s reservation profit is given by (18) and the price must satisfy (19). Maximizing the purchaser’s payoff function again yields

$$y^* = K - \frac{(A - V)}{2b} \quad (66)$$

but in this case, the price is

$$T_{FB} = \frac{V^2 - c\left(2A - c - 4bK\right)}{4b}. \quad (67)$$

In this case, the purchaser’s profit, as a function of $K$, is computed as:

$$V_y^* - T^* = \frac{1}{4b}(V - c)(V - 2A + c + 4bK). \quad (68)$$

**Proof of Proposition 4**

This proposition characterizes the values, $K^c$, $K^{SC}$, and $K^{FB}$. No product will be sold to the purchaser under the No Information contract if $K \leq K^c = \frac{(A + 4b\bar{K} - V)}{6b}$. Given the values of $(y^{NI}, T^{NI})$ in Proposition 1, the expected payoff to the supplier is:

$$\Pi_s\left(y^{NI}, T^{NI}\right) = \frac{A^2 + b^2\left(12K\bar{K} - 8\bar{K}^2 - 9K^2\right) + A\left(3bK + 2b\bar{K} - 2V\right) + V^2 + b\left(6KV - 2V\bar{K} - 9cK\right)}{9b}. \quad (69)$$

If $K \leq K^1$, the supplier is subject to lost profits of

$$\frac{(A + 4b\bar{K} - V)(5A - 6c - 4b\bar{K} + V)}{6b} \quad (70)$$
if the offer is accepted. The profits in (69) just cover the lost profits in (70)

\[ K \geq K^c = \frac{(A + 4b\bar{K} - V)}{6b}. \]

QED

No product will be sold to the purchaser under the Supply Chain contract

if \( K \leq K^{sc} = \frac{A + 2b\bar{K} - V}{4b} \). The value of \( y^{sc} \) in (14) is \( 2K - \bar{K} - \frac{A-V}{2b} \). This value is zero if \( K = K^{sc} \) and would be negative for any value of \( K < K^{sc} \). QED

No product will be sold to the purchaser under the Full Information contract

if \( K \leq K^{fb} = \frac{(A-V)}{2b} \). Since the value of \( y^{fb} \) in (20) is \( K - \frac{(A-V)}{2b} \), this value is negative if \( K < K^{fb} \).

QED

**Proof of Proposition 5**

For all values of \( K \), \( y^{fb} > y^{sc} \) and \( y^{fb} \geq y^{ni} \). \( y^{fb} = y^{ni} \) when \( K = K^{NI} \). In addition, there exists a unique capacity level, \( K^{NI}^{sc} \), such that for all \( K \in (K^{NI}, K^{NI}^{sc}) \), \( y^{NI} > y^{sc} \) and for all \( K < K^{NI} \) and \( K > K^{NI}^{sc} \), \( y^{NI} < y^{sc} \). Finally, the offer of \( y^{NI} \) is greater than the expected quantity sold under the Full Information contract.

First we show that \( y^{fb} > y^{ni} \) and \( y^{fb} > y^{sc} \). From (20) (and (22)) \( y^{fb} = K - \frac{(A-V)}{2b} \), from (6), \( y^{ni} = \frac{1}{3b} (V - A + 2b\bar{K}) \), and from (14), \( y^{sc} = 2K - \bar{K} - \frac{A-V}{2b} \). From inspection, it is clear that \( y^{fb} - y^{sc} = \bar{K} - K \) so \( y^{fb} > y^{ni} \). Recall that no product will be supplied to the purchaser for \( K < K^c = \frac{(A + 4b\bar{K} - V)}{6b} \), so for \( K < K^c \), \( y^{ni} = 0 \). At
\( K = K^c, \ y^{FB} = y^{NI} = \frac{1}{3b} \left( V - A + 2bK \right) \), and for \( K > K^c \), \( y^{FB} \) is increasing in \( K \) and therefore greater than \( y^{NI} \) since \( y^{NI} \) is fixed in \( K \). Clearly, for \( K < K^c, 0 = y^{NI} < y^{SC} \).

Note that \( y^{SC} \) is linearly increasing in \( K \) and is equal to \( y^{NI} \) at \( K = \frac{A + 10bK - V}{12b} \).

Finally, the expected sales of \( y \) under the Full Information contract is computed as

\[
E[y^{FB}] = \int_{(A-V)/2b}^{K} \frac{1}{K} \left( k - \frac{(A-V)}{2b} \right) dk = \frac{(A - 2bK + V)^2}{8b^2K}. \]

This value is less than \( y^{NI} = \frac{1}{3b} \left( V - A + 2bK \right) \) as long as \( V > A - 2bK \), which is true by assumption.

**Proof of Proposition 6**

We begin by comparing the expected payoff to the purchaser under the No Information and Supply Chain contracts. The expected payoff to the purchaser under the No Information contract is computed as

\[
\Pi_p^{NI} = \int_{(A-\frac{2bK-V}{6b})}^{K} g[k] \left( Vy^{NI} - T^{NI} \right) dk \]

where \( y^{NI} \) and \( T^{NI} \) are given in (6) and (7) respectively. This expected payoff simplifies to:

\[
\Pi_p^{NI} = \frac{(V + 2bK - A)^3}{54b^2K}. \quad (71)
\]

The expected payoff to the purchaser under the Supply Chain contract is computed as

\[
\Pi_p^{SC} = \int_{(A+\frac{2bK-V}{4b})}^{K} g[k] \left( Vy^{SC} - T^{SC} \right) dk \]

where \( y^{SC} \) and \( T^{SC} \) are given in (14) and (15) respectively. This expected payoff simplifies to:

\[
\Pi_p^{SC} = \frac{(V + 2bK - A)^3}{48b^2K}. \quad (72)
\]
Clearly, by inspection, (72) is greater than (71) because the numerators are identical, but the denominator of (71) is greater. As a result, \( \Pi^{SC}_p > \Pi^{NI}_p \). The expected payoff to the purchaser under the Full Information contract is computed as

\[
\Pi^F_B = \int_{(c-V)/(c-V)}^{(c-\bar{V})/2b} g(k)(V^F_B - T_1^F_B)dk + \int_{(c-\bar{V})/2b}^{\bar{V}} g(k)(V^F_B - T^F_B)dk \quad \text{where} \quad y^F_B \text{ is given in (20), } T_1^F_B \text{ is given in (21), and } T_2^F_B \text{ is given in (23). This expression simplifies to}
\]

\[
\Pi^F_B = \frac{1}{24b^2\bar{K}}\left((V-c)^3 + 3(A-c-2b\bar{K})(V-c)(A-2b\bar{K}-V)\right). \tag{73}
\]

Comparing (73) and (72), \( \Pi^F_B > \Pi^{SC}_p \) whenever

\[
\Delta = \left(A-2b\bar{K}-V\right)^3 + 2\left((V-c)^3 + 3(A-c-2b\bar{K})(V-c)(A-2b\bar{K}-V)\right) > 0. \tag{74}
\]

Computing \( \frac{\partial \Delta}{\partial V} \), we observe that the expression in (74) is increasing in \( V \). We compute the value of \( V \) for which \( \Delta \) is zero to be:

\[
V > 2^{1/3} c + \left(1 - 2^{1/3}\right)\left(A - 2b\bar{K}\right). \tag{75}
\]

Hence, for \( V \) large enough, the purchaser obtains a strictly higher expected payoff under the Full Information contract than under the Supply Chain contract. Of course, this condition is sufficient but not necessary. It is sufficient to note that the two contracts, from the perspective of the purchaser, are identical, except that the Supply Chain contract has an additional constraint. That also implies that the agent, under the Supply Chain contract, obtains information rents that it does not obtain under the Full Information contract.