F371 Exam 1 High-Priority Practice Problems

Three types of problems are especially important to understand for Exam 1. These are:

- Rate mismatch problems
- Amortization problems
- Perpetuity problems

Here is an example of each type.

**Rate Mismatch Problem**

*When making computations for annuities, all factors must be expressed using the same time period. If payments are monthly, for example, then N must be the number of months, and I/Y must be a rate per month. Same for any other time period: everything must match the payment period.*

*Example:*

A company borrowed $150,000. The interest rate on the loan is 9.12% compounded monthly. But the company plans to make quarterly payments, at the end of each quarter, for six years. What will be the amount of the quarterly payment?

Payments are quarterly, so we need a rate per quarter. APR = 9.12% compounded quarterly, so \( R = \frac{9.12}{12} = 0.76\% \) per month. Convert that to a rate per quarter.

\[
\left(1 + r_{\text{quarterly}}\right)^4 = \left(1 + r_{\text{monthly}}\right)^{12} = 1.0076^{12} = 1.0951
\]

\[
1 + r_{\text{quarterly}} = 1.0951^{\frac{1}{4}} = 1.022974
\]

\( R \text{ per quarter} = 2.2974\% \)

Now solve for the quarterly payment.

<table>
<thead>
<tr>
<th>N</th>
<th>I/Y</th>
<th>PV</th>
<th>PMT</th>
<th>FV</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>2.2974</td>
<td>150,000</td>
<td>?</td>
<td>0</td>
</tr>
</tbody>
</table>

The payment amount is $8,200.33.
**Amortization Problem**

*With an amortized loan, part of each payment goes to pay off the loan (i.e., to pay off the principal). Since the amount still owed (the principal) goes down with each payment, the amount of interest that comes out of each payment is different every time.*

**Example:**
A $5,000 consumer loan carries an interest rate of 9.48% compounded monthly. The loan will be paid off with five years of monthly payments, paid at the end of each month. How much of the second payment will go toward paying off the principal?

First, solve for the monthly payment. From the APR, compute R per month = 9.48 / 12 = 0.79% per month.

<table>
<thead>
<tr>
<th>N</th>
<th>I/Y</th>
<th>PV</th>
<th>PMT</th>
<th>FV</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>0.79</td>
<td>5,000</td>
<td>?</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-104.96</td>
<td></td>
</tr>
</tbody>
</table>

From the first payment, interest will be figured on the amount owed that period, which was $5,000. Interest is $5000 x .0079 = $39.50. The remainder of the payment amount, which is $65.46, will pay down the principal. The ending balance after the first payment is $4,934.54.

That becomes the beginning balance for the second period. Payment amount is the same $104.96. Interest in the second period is figured on the amount owed that period, which was $4,934.54. Interest = 4,934.45 x .0079 = $38.98.

The amount from the second payment which goes toward the principal is the remainder of the payment: $104.96 – $38.98 in interest = $65.98 toward the principal.

The amortization table for the first two payments looks like this:

<table>
<thead>
<tr>
<th>Beginning Bal</th>
<th>Payment</th>
<th>Interest</th>
<th>Principal</th>
<th>Ending Bal</th>
</tr>
</thead>
<tbody>
<tr>
<td>5,000.00</td>
<td>104.96</td>
<td>39.50</td>
<td>65.46</td>
<td>4,934.54</td>
</tr>
<tr>
<td>4,934.54</td>
<td>104.96</td>
<td>38.98</td>
<td>65.98</td>
<td>4,868.56</td>
</tr>
</tbody>
</table>

**Perpetuity Problem**

*A perpetuity is an infinite series of cash flows. There is no end, so there is no N to put into a TVM problem. If the amount of the cash flows in the perpetuity is the same in every period, there is a formula to compute the present value of the perpetuity. The formula is:*

\[
P_{\text{per}} = \frac{CF_1}{r}
\]
The key thing to note about this formula is that it gives the present value as of the perpetuity’s Time Zero.

If the perpetuity begins at some point in the future, it’s important to draw the perpetuity’s timeline. That way you can determine where the perpetuity’s Time Zero occurs. And that’s where the perp’s PV will be located on the timeline.

Example:
A certain security will pay $84 per year forever. But the first payment won’t be made until the end of five years from now. At a required rate of return of 7%, what should be the price today of that security?

The timeline looks like this:

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | ...
|---|---|---|---|---|---|---|---|---|---|
| PV | ? | 84 | 84 | 84 | 84 | 84 | 84 | 84 | ...

Now draw the perpetuity’s timeline. Its first cash flow occurs at our Time 5, so that’s Time 1 of the perp. The timeline now looks like this, with the perpetuity timeline drawn in above:

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | ...
|---|---|---|---|---|---|---|---|---|---|
| PV | ? | 84 | 84 | 84 | 84 | 84 | 84 | 84 | ...

Therefore, the Time Zero of the perpetuity is at our Time 4.

Use the perpetuity formula to find the present value as of the perp's Time Zero.

\[
P_{\text{perp}} = \frac{C}{r}
\]

\[
= \frac{84}{.07}
\]

\[
= 1,200.00
\]

That $1,200.00 occurs at the Time Zero of the perpetuity, which is our Time 4:

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | ...
|---|---|---|---|---|---|---|---|---|---|
| PV | ? | 84 | 84 | 84 | 84 | 84 | 84 | 84 | ...
| PV | 1,200

To find the present value today, at our Time Zero, discount the $1,200 for FOUR periods (not five!) at a 7% discount rate. Discount it four periods, because it is in effect a Time 4 cash flow on our timeline – which is Time Zero of the perpetuity.

\[
PV = \frac{1200}{1.07^4}
\]

\[
= 915.47
\]

The present value today is $915.47.